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Full-State Feedback Control Design for Shape Formation using Linear Quadratic Regulator

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ABSTRACT

This study investigated the capability of a group of agents to form a desired shape formation by designing the feedback control using a linear quadratic regulator. In real application, the state condition of agents may change due to some particular problems such as a slow input response. In order to compensate for the problem that affects agent-to-agent coordination, a robust regulator was implemented into the formation algorithm. In this study, a linear quadratic regulator as the full-state feedback of robust regulator method for shape formation was considered. The result showed that a group of agents can form the desired shape (square) formation with a modification of the trajectory shape of each agent. The results were validated through numerical experiments.

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1. INTRODUCTION

Formation control is an important aspect of agent coordination. It has attracted tremendous attention from researchers due to its benefit in many applications such as an exploration of group robots (Burgard et al., 2005, 2000), surveillance (Roman-Ballesteros & Pfeiffer, 2006), unmanned quadrotor aerial formation (Fikri et al., 2018; Hou & Fantoni, 2016), and spacecraft formation guidance (Beard et al., 2001; Beard & Hadaegh, 1999).

The requirement to achieve a desired formation shape has been investigated by considering the relativity of the position and velocity of the agent. Several works have been conducted such as individual motion in nature (Fax & Murray, 2004), decentralized schemes (Lafferriere et al., 2005), and potential field (Zavlanos & Pappas, 2007) to perform the shape formation control.

In other researches, the feedback scheme method (Fax & Murray, 2004; Lafferriere et al., 2005) has been proposed. This scheme associated with a directed graph has been investigated by (Liu et al., 2019; Wu et al., 2011). However, the proposed systems are less robust when the state matrix does not follow the setting (Lafferriere et al., 2005). To keep the formation in the desired form, it requires the change of the state condition to be sustained.

In this paper, we propose a linear quadratic regulator, LQR, as the feedback control method to improve the system robustness toward the change of state feedback. The proposed method enables the agent to move following a different trajectory to create a square shape formation. The paper is outlined as follow. Section 2 provides the method of how the graph theory can be implemented into a feedback control system and also the employed agents model. Moreover, it also explains how the full-state feedback is formulated. After the formulation is achieved, we validate the state feedback with numerical analysis to prove its capability. Then, a discussion on the numerical analysis result is given in section 3. We close this paper with a conclusion and future work in section 4.

2. METHODOLOGY

In this section, the method on how the graph theory can be implemented into a feedback control is delivered. The content based on the introduction of graph theory, problem statement, and the design of Linear Quadratic Regulator (LQR) feedback control.

2.1. Graph Theory

The graph is used to model the communication among agents. A graph $\mathcal{G}$ is defined by a pair of a finite set of nodes and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, or $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. An edge is represented by an ordered pair of nodes, i.e. $e_q = (j, i) \in \mathcal{E}$. The pair $(j, i) \in \mathcal{E}$ if node j points towards node i. The set of neighbors of node i is $\mathcal{N}_i := \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, i \neq j\}$. Below is the important definition of the graph.

Definition 1: (Lewis, F., Zhang, H., Hengster-Movric, K., Das, 2014)

A graph is said to be an *undirected* graph if $a_{ij} = a_{ji}, \forall\, i, j$ and also if it is a bidirectional and the weights of the edge (v_i, v_j) and (v_j, v_i) are the same.

The adjacency matrix associated with the graph $\mathcal{G}$ is $[\mathcal{A}_{ij}] = [a_{ij}]$, where:

$$a_{ij} = \begin{cases} 0, & i = j \\ 1, & j \in \mathcal{N}_i \end{cases} \tag{1}$$

To apply the graph to our model, the *Laplacian matrix* is used. We define $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ s.t $l_{ij} = \Sigma_{j \in \mathcal{N}_i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. Agents need to exchange the information, where the Laplacian matrix follows (Mesbahi and Egerstedt, 2010). The Laplacian matrix can be also expressed as:

$$\mathcal{L}(\mathcal{G}) = \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}). \tag{2}$$

Here, $\mathcal{D}(G)$ denotes the degree matrix of graph $\mathcal{G}$. Thus, the following result will be used in this paper.

Lemma 1: (Ren & Beard, 2005)

Let $\mathcal{L}$ be the *Laplacian* matrix associated with undirected graph $\mathcal{G}$. $\mathcal{L}$ has the following properties:

i. 0 is an eigenvalue of $\mathcal{L}$ with eigenvector $\mathbf{1}_N$,

ii. The nonzero eigenvalues of $\mathcal{L}$ have a positive real part,

iii. 0 is a simple eigenvalue of $\mathcal{L}$ if and only if graph $\mathcal{G}$ is connected.

2.2. Problem Statement

In this work, we consider N agents given by the following:

$$\dot{x}_i(t) = \boldsymbol{A}x_i(t) + \boldsymbol{B}u_i(t), i = 1, \dots, N \quad (3)$$

where $x_i \in \mathbb{R}^{2n}$ is a state of agent i, $u_i \in \mathbb{R}^n$ is the control input of agent i for $i = \{1, \dots, N\}$. Meanwhile, $\boldsymbol{A}$ and $\boldsymbol{B}$ are the state and input matrices of appropriate size.

The state x_i consists of two elements, $p_i \in \mathbb{R}^n$ and $v_i \in \mathbb{R}^n$, where p_i and v_i are position and velocity respectively. Then, $x_i = [p_i' \ v_i']'$ or

$$x_i = p_i \otimes \begin{pmatrix}1\\0\end{pmatrix} + v_i \otimes \begin{pmatrix}0\\1\end{pmatrix} \quad (4)$$

where $\otimes$ denotes the Kronecker product. The Kronecker product corresponds with the configuration state of the individual agent.

We assume the state matrix to be of the form:

$$\boldsymbol{A} = \operatorname{diag}(a_1, \dots, a_n) \quad (5)$$

where $a_i = \begin{bmatrix}0 & a_i\\ -a_i & 0\end{bmatrix}$ for $i = \{1, \dots, n\}$, and

$$\boldsymbol{B} = \boldsymbol{I}_N \otimes \begin{pmatrix}0\\1\end{pmatrix} \quad (6)$$

Definition 3: Let the formation denotes by vector

$$h = h_p \otimes \begin{pmatrix}1\\0\end{pmatrix} \in \mathbb{R}^{2nN} \quad (7)$$

Where $h_p = \left[h_{p1}', \dots, h_{pN}'\right]'$. The formation h is reached if and only if $\lim_{t \to \infty} \left(p_i(t) - p_j(t)\right) = h_{pi} - h_{pj}$ for all i, j. This is illustrated in **Figure 1**.

The goal of the output feedback is to steer the agents to the desired formation. We define the error of output function z_i based on the average of relative displacement of the neighboring agents.

$$z_i = (x_i - h_i) - \frac{1}{\mathcal{N}_i} \sum_{j \in \mathcal{N}_i} (x_j - h_j) \quad (8)$$

where $i = 1, \dots, N$. Let $z = [z_1', \cdots, z_N']'$.

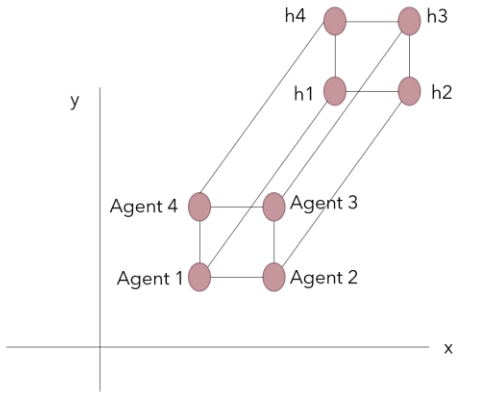

Figure 1. Illustration of agents in a square formation

We can write the overall system as:

$$\dot{x} = \bar{\boldsymbol{A}}x + \bar{\boldsymbol{B}}u \quad (9)$$

$$z = L(x - h) \quad (10)$$

with $\bar{\boldsymbol{A}} = \boldsymbol{I}_N \otimes \boldsymbol{A}$ and $\bar{\boldsymbol{B}} = \boldsymbol{I}_N \otimes \boldsymbol{B}$.

Remark 1: By referring to Definition 3 and (10), the agents can be said are in formation if only if z = 0.

Problem 1: The objective in this paper is to design control input u_i by using the output vector z_i such that the formation h is achieved.

2.3. LQR Feedback Control

We consider a single system of the optimal feedback control using linear quadratic regulator (LQR), where the system is described by:

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u \quad (11)$$

where x is state, u is control input. To stabilize the system (11), the control input is designed to minimize the cost function:

$$\boldsymbol{J} = \int_0^\infty (x'\boldsymbol{Q}x + u'\boldsymbol{R}u)\, dt \quad (12)$$

Here, $\boldsymbol{Q}$ is $n \times n$ symmetric positive semidefinite matrix $(\boldsymbol{Q} \geq 0)$, and $\boldsymbol{R}$ is $m \times m$ symmetric positive definite matrix $(\boldsymbol{R} > 0$. Thus, the optimization can be written as

$$\underset{u\in\mathbb{R}^m}{\text{minimize}}\ \boldsymbol{J}$$
$$s.t.\ \ \dot{x}_i(t) = \boldsymbol{A}x_i(t) + \boldsymbol{B}u_i(t) \quad (13)$$

The control input is assumed to have the form

$$u = -\boldsymbol{K}x \quad (14)$$

To find $\boldsymbol{K}$**,** we define it as:

$$\boldsymbol{K} = \boldsymbol{T}^{-1}(\boldsymbol{T}^*)^{-1}\boldsymbol{B}^*\boldsymbol{P} = \boldsymbol{R}^{-1}\boldsymbol{B}^*\boldsymbol{P} \quad (15)$$

where equation (15) gives the optimal control of $\boldsymbol{K}$**.** Thus the optimal control that we can obtain is defined by:

$$u(t) = -\boldsymbol{K}x = \boldsymbol{R}^{-1}\boldsymbol{B}^*\boldsymbol{P}x(t) \quad (16)$$

where, $\boldsymbol{P}$ in (15) follows the solution to Algebraic Riccati Equation (ARE) of $n \times n$ symmetric which defines by:

$$\boldsymbol{A}^*\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^*\boldsymbol{P} + \boldsymbol{Q} = 0 \quad (17)$$

Then, LQR in the systems can be achieved by following several steps:

1. Given $\boldsymbol{A}$**,** and $\boldsymbol{B}$ the state matrix
2. Select $\boldsymbol{Q}$**,** and $\boldsymbol{R}$ for regulator matrix
3. Solve using ARE to find $\boldsymbol{P}$**.**
4. Compute $\boldsymbol{K} = \boldsymbol{R}^{-1}\boldsymbol{B}^*\boldsymbol{P}$, and
5. Choose the solution of $\boldsymbol{K}$ which yields the state of the systems.

3. RESULT AND DISCUSSION

To solve problem 1, the following is the proposed control input.

$$u_i = \boldsymbol{F}\sum l_{ij}z_j \quad (18)$$

where $z_j = x_j - h_j$, and $\boldsymbol{F}$ is a feedback matrix. Thus, the closed-loop system for agent i can be written as:

$$\dot{x}_i(t) = \boldsymbol{A}x_i(t) + \boldsymbol{B}\boldsymbol{F}\sum l_{ij}z_j \quad (19)$$

Meanwhile, the overall closed-loop system can be written as:

$$\dot{x} = \overline{\boldsymbol{A}}x + \overline{\boldsymbol{B}}\overline{\boldsymbol{F}}L(\mathrm{x} - \mathrm{h}) \quad (20)$$

where $\overline{\boldsymbol{F}} = \boldsymbol{I}_N \otimes \boldsymbol{F}$. Following the matrix states of $\boldsymbol{A}, \boldsymbol{B}$, and L. The following is the main result of this paper.

Proposition 1. Consider N agents given by (3) with the control input given by (11). Suppose the graph $\mathcal{G}$ is connected. The formation h is achieved if and only if $A + \lambda_2 BF$ is Hurwitz, where λ_2 is the smallest nonzero eigenvalue of L.

To show the above proposition holds, we consider the following problem example. We conduct the numerical experiment by given the agent $N = 4$ with consideration a square is the targeted shape formation. Here, several parameters we include in our numerical experiment. First, we state,

$$\boldsymbol{A} = \begin{bmatrix} 0 & \alpha_i & 0 & 0 \\ -\alpha_i & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_i \\ 0 & 0 & -\alpha_i & 0 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The value in the matrix A will be changed to know the capability of the proposed method to form a stable formation. Then, to develop the square formation which is stable in the numerica analysis we follow (7) by giving a set of h_p by,

$$\boldsymbol{h}_p = \begin{bmatrix} \mathrm{r} * \cos\left(\frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \sin\left(\frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \cos\left(\mathrm{pi} - \frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \sin\left(\mathrm{pi} - \frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \cos\left(\mathrm{pi} + \frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \sin\left(\mathrm{pi} + \frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \cos\left(2 * \mathrm{pi} - \frac{\mathrm{pi}}{4}\right) \\ \mathrm{r} * \sin\left(2 * \mathrm{pi} - \frac{\mathrm{pi}}{4}\right) \end{bmatrix}$$

The feedback control design using LQR which is shown in the (13) we set $\boldsymbol{Q}$, and $\boldsymbol{R}$ matrices as:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To form a square formation, the adjacency and the degree matrix of the formation is set to be

$$\mathcal{D}(\mathcal{G}) = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

And

$$\mathcal{A}(\mathcal{G}) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Then, the graph established from the degree matrix and adjacency matrix is shown in **Figure 2**.

The graph is set not automatically. In this study, to form the graph is based on the degree matrix and the adjacency matrix. Thus, Eigenvalue of the Laplacian matrix is,

$$spec\{L\} = \begin{bmatrix} 0.000 \\ 2.000 \\ 2.000 \\ 2.000 \end{bmatrix}$$

Thus, $\lambda_2 = 2$. By Proposition 1, to reach the formation, we must ensure that the eigenvalue of $A + \lambda_2 BF$ is stable. Using the LQR method explained in section 2 and the Q and R matrices given above, the feedback matrix F is:

$$\boldsymbol{F} = \boldsymbol{diag}\{\boldsymbol{F_1}, \boldsymbol{F_1}\}$$

$$F_1 = \begin{bmatrix} -315.23 & -317.22 \\ 0 & 0 \end{bmatrix}$$

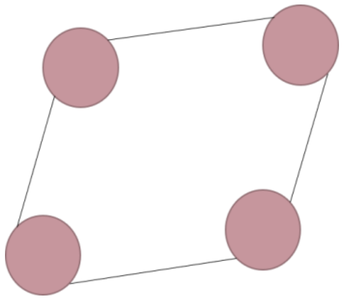

Figure 2. Illustration of graph topological network

Then, we have the eigenvalue of $\boldsymbol{A + \lambda_2 BF}$ given in the array below.

$$spec\{A + \lambda_2 BF\} = \begin{bmatrix} -0.9969 \\ -633.4492 \\ -0.9969 \\ -633.4492 \\ -633.4492 \\ -0.9969 \\ -0.9969 \\ -633.4492 \\ -633.4492 \\ -0.9969 \\ -0.9969 \\ -633.4492 \\ -633.4492 \\ -0.9969 \\ -0.9969 \\ -633.4492 \\ -633.4492 \\ -0.9969 \end{bmatrix}$$

The $A + \lambda_2 BF$ is Hurwitz. Then, the square shape formation can be achieved by following the value of h_p. The result of the setting is shown in **Figure 3**.

The initial position of the four agents is set by choosing a random position. As we can see, the agents form a square formation while they are also tracking an elliptical trajectory at their respective position. The trajectory is a function of the initial state. We show that agents track a different trajectory by setting different initial states. This result is shown in **Figure 4**.

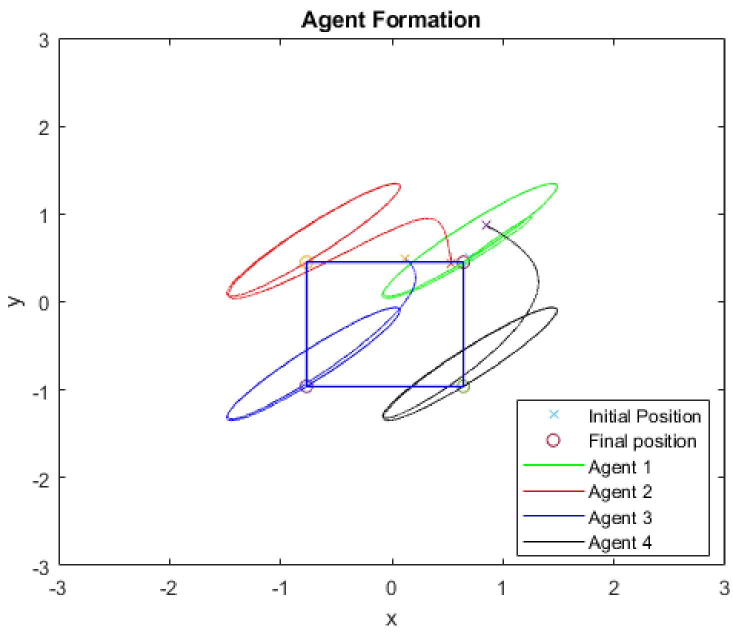

Figure 3. The formation established by four agents with a random initial position.

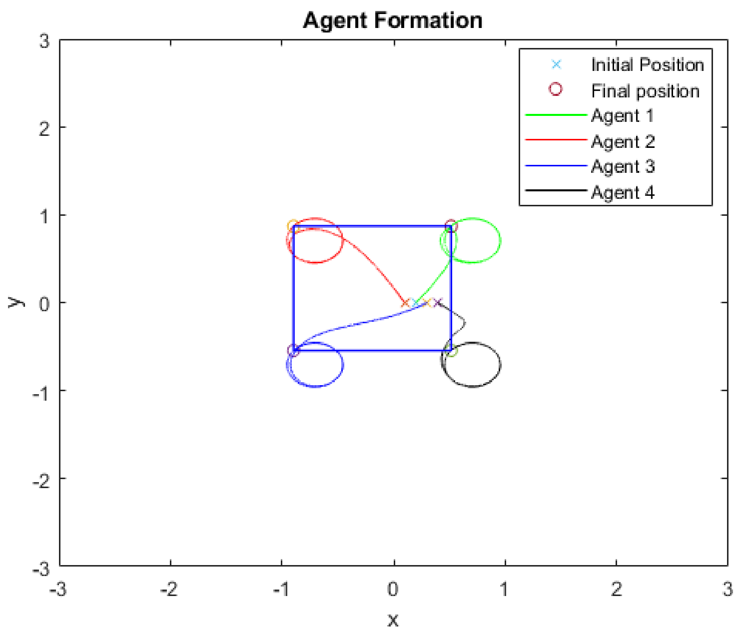

Figure 4. The formation established by four agents with a non-random initial position.

Furthermore, we show the result when at least one of the eigenvalue of $A + \lambda_2 BF$ has a positive real part. This is done by using $F = -diag\{F_1, F_1\}$. The result yields some of the eigenvalues have positive real parts and the formation is not achieved. In this study, the result of unachievable formation is not discussed.

4. CONCLUSION

We have successfully designed a feedback control for multi-agent systems to establish a square shape formation using Linear Quadratic Regulator (LQR). The output control is established based on the control input that is designed through LQR to compensate the stability of the formation, where the design gives a result of a stable eigenvalue. Thus, the square shape formation can be achieved. In addition, the trajectory shape of the agents also can be modified, where the stability of the trajectory while creating a shape fomation is affected by the LQR. In the future, investigation on the shape formation control using switching graph will be considered.

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REFERENCES

Beard, R. W., & Hadaegh, F. Y. (1999). Fuel optimization for unconstrained rotation of spacecraft formations. *Journal of the Astronautical Sciences*, *47*(3–4). https://doi.org/10.1007/bf03546203

Beard, R. W., Lawton, J., & Hadaegh, F. Y. (2001). A coordination architecture for spacecraft formation control. *IEEE Transactions on Control Systems Technology*, *9*(6). https://doi.org/10.1109/87.960341

Burgard, W., Moors, M., Fox, D., Simmons, R., & Thrun, S. (2000). Collaborative multi-robot exploration. *Proceedings - IEEE International Conference on Robotics and Automation*, *1*. https://doi.org/10.1109/robot.2000.844100

Burgard, W., Moors, M., Stachniss, C., & Schneider, F. E. (2005). Coordinated multi-robot exploration. *IEEE Transactions on Robotics*, *21*(3). https://doi.org/10.1109/TRO.2004.839232

Fax, J. A., & Murray, R. M. (2004). Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control*, *49*(9). https://doi.org/10.1109/TAC.2004.834433

Fikri, M., Ashari, A., Dharmawan, A., & Alldino, B. (2018). An Overview of Fundamental Step Using Wi-Fi Communication for Flight Formation Quadrotors. *Proceedings - 2018 4th International Conference on Science and Technology, ICST 2018*. https://doi.org/10.1109/ICSTC.2018.8528700

Hou, Z., & Fantoni, I. (2016). Leader-follower formation saturated control for multiple quadrotors with switching topology. *2015 Workshop on Research, Education and Development of Unmanned Aerial Systems, RED-UAS 2015*. https://doi.org/10.1109/RED-UAS.2015.7440985

Lafferriere, G., Williams, A., Caughman, J., & Veerman, J. J. P. (2005). Decentralized control of vehicle formations. *Systems and Control Letters*, *54*(9). https://doi.org/10.1016/j.sysconle.2005.02.004

Lewis, F., Zhang, H., Hengster-Movric, K., Das, A. (2014). *Cooperative Control of Multi-Agent Systems - Optimal and Adaptive Design Approaches*. Springer-Verlag.

Liu, T., De Queiroz, M., Zhang, P., & Khaledyan, M. (2019). Directed formation control of n planar agents with distance and area constraints. *Proceedings of the American Control*

Conference, *2019-July*. https://doi.org/10.23919/acc.2019.8815084

Ren, W., & Beard, R. W. (2005). Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control*, *50*(5). https://doi.org/10.1109/TAC.2005.846556

Roman-Ballesteros, I., & Pfeiffer, C. F. (2006). A framework for cooperative multi-robot surveillance tasks. *Proceedings - Electronics, Robotics and Automotive Mechanics Conference, CERMA 2006*, *2*. https://doi.org/10.1109/CERMA.2006.3

Wu, B., Wang, D., & Poh, E. K. (2011). Decentralized robust adaptive control for attitude synchronization under directed communication topology. *Journal of Guidance, Control, and Dynamics*, *34*(4). https://doi.org/10.2514/1.50189

Zavlanos, M. M., & Pappas, G. J. (2007). Potential fields for maintaining connectivity of mobile networks. *IEEE Transactions on Robotics*, *23*(4). https://doi.org/10.1109/TRO.2007.900642